HQC: Hamming Quasi-Cyclic

An IND-CCA2 Code-based Public Key Encryption Scheme

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https://pqc-hqc.org

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Outline

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HQC Classification / Design Rationale

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Important features:

- IND-CPA code-based PKE
- Reduction to a well-known and difficult problem:

  Decoding random quasi-cyclic codes
- No hidden trap in the code
- Efficient decoding (BCH + repetition code)
- Accurate failure rate

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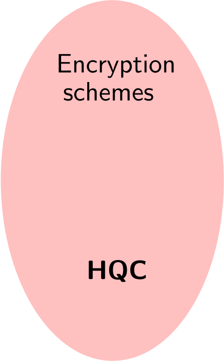

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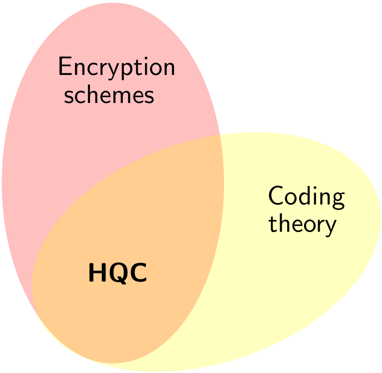

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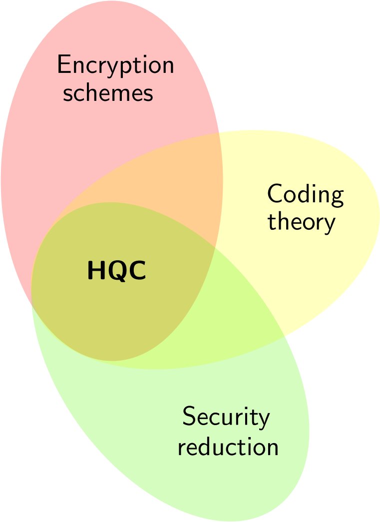

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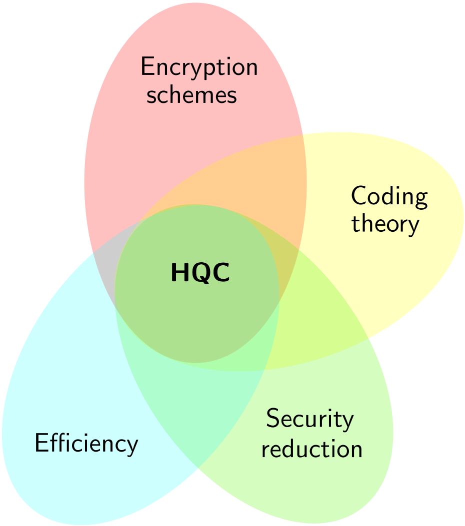

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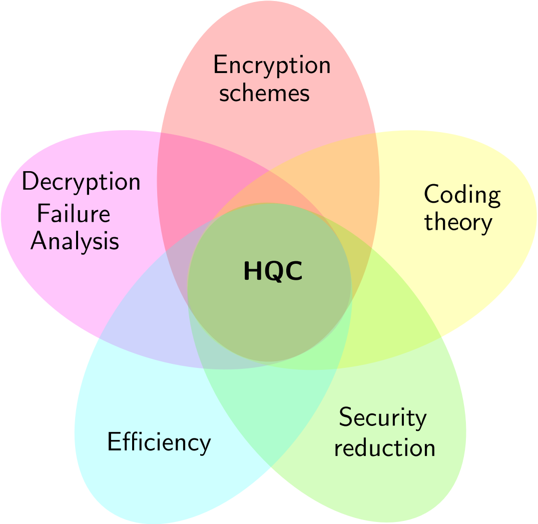

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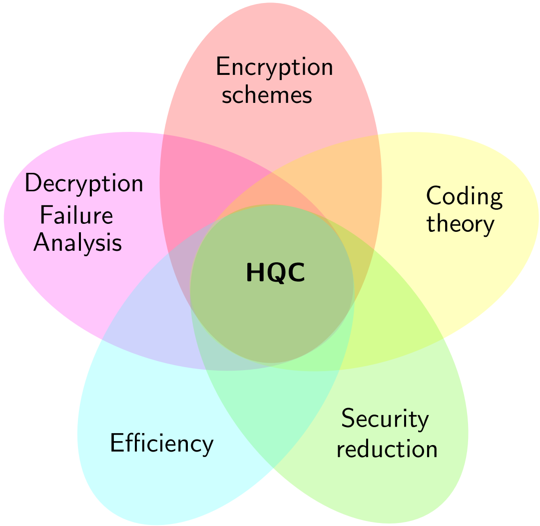

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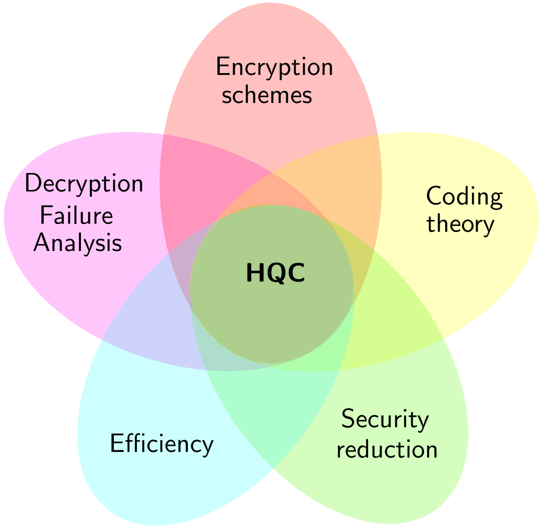

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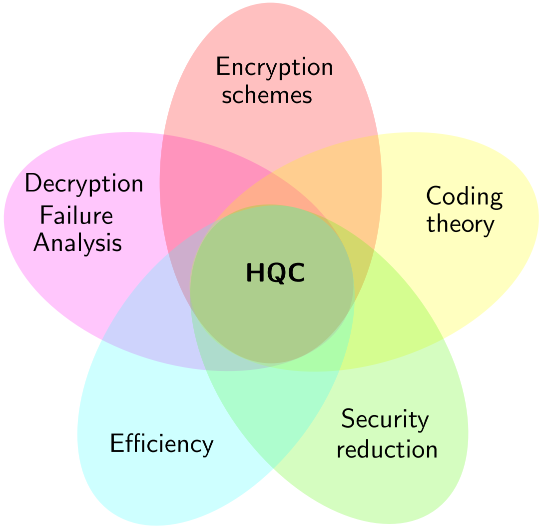

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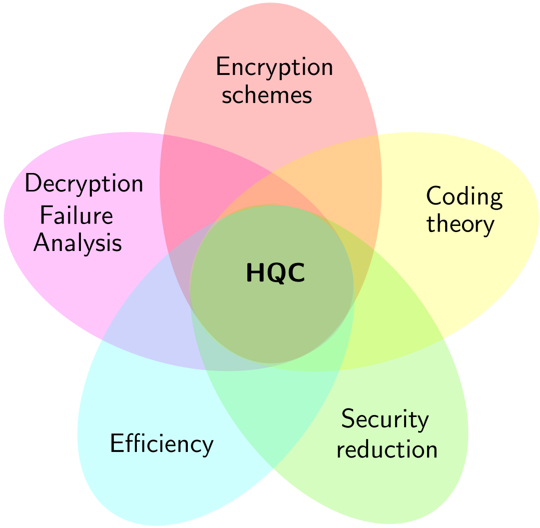

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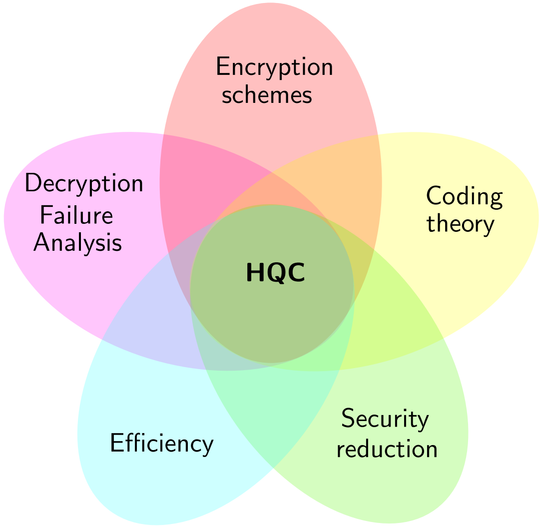

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HQC Encryption Scheme [ABD+18]

Encryption scheme in **H**amming metric, using **Q**uasi-**C**yclic Codes

- Notation: Secret data - Public data - One-time Randomness
- $\mathbf{G}$ is the generator matrix of some public code $\mathcal{C}$
- $\mathcal{S}_w^n(\mathbb{F}_2) = \{\mathbf{x} \in \mathbb{F}_2^n$ such that $\omega(\mathbf{x}) = w\}$

Alice		Bob
$\mathrm{seed}_{\mathbf{h}} \xleftarrow{\$} \{0,1\}^{\lambda}$, $\mathbf{h} \xleftarrow{\mathrm{seed}_{\mathbf{h}}} \mathbb{F}_2^n$ $\mathbf{x}, \mathbf{y} \xleftarrow{\$} \mathcal{S}_w^n(\mathbb{F}_2)$, $\mathbf{s} \leftarrow \mathbf{x} + \mathbf{h}\mathbf{y}$	$\xrightarrow{\mathrm{seed}_{\mathbf{h}}, \mathbf{s}}$	
	$\xleftarrow{\mathbf{u}, \mathbf{v}}$	$\mathbf{r}_1, \mathbf{r}_2 \xleftarrow{\$} \mathcal{S}_w^n(\mathbb{F}_2)$, $\mathbf{e} \xleftarrow{\$} \mathcal{S}_w^n(\mathbb{F}_2)$ $\mathbf{u} \leftarrow \mathbf{r}_1 + \mathbf{h}\mathbf{r}_2$, $\mathbf{v} \leftarrow \mathbf{m}\mathbf{G} + \mathbf{s}\mathbf{r}_2 + \mathbf{e}$
$\mathbf{m} \leftarrow \mathcal{C}.\mathrm{Decode}(\mathbf{v} - \mathbf{u}\mathbf{y})$		

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HQC Instantiation

HQC is a generic framework to build efficient and secure code-based cryptosystems

Proposed instantiation:

- BCH codes tensored with repetition codes
  - Efficient decoding
  - Accurate DFR estimates

Time in ms
Intel® Core™ i7-4770 CPU @ 3.4GHz

Instance	KeyGen	Encaps	Decaps
Strength 1	0.17-0.19	0.36-0.40	0.57-0.63
Strength 3	0.37-0.43	0.77-0.89	1.13-1.28
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(Number of cycles available in supporting documentation)

Theorem

HQC is IND-CPA under 2-DQCSD and 3-DQCSD.

2-Decisional Quasi-Cyclic Syndrome Decoding and 3-DQCSD Problems

Instance: $\mathbf{h}, \mathbf{s} \in \mathbb{F}_2^n$

Decide: $\exists ?(\mathbf{x}, \mathbf{y}) \in \mathcal{S}_w^n(\mathbb{F}_2)$ s.t. $\mathbf{s} = \begin{pmatrix} \mathbf{I}_n & \mathbf{h} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}$

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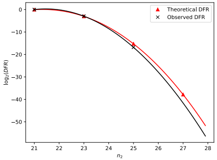

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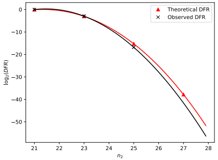

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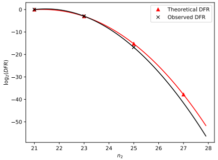

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Parameters

All sizes in **bytes**

NIST Cat.	Instance	pk size sizeof($\mathbf{h}, \mathbf{s}$) (sizeof(seed$_{\mathbf{h}}$, $\mathbf{s}$))	sk size sizeof($\mathbf{x}, \mathbf{y}$) (sizeof(seed$_{sk}$))	ct size	DFR
1	Basic-I	5,558 (2,819)	252 (40)	5,622	2^{-64}
	Basic-III	6,170 (3,125)	252 (40)	6,234	2^{-128}
3	Advanced-I	10,150 (5,115)	404 (40)	10,214	2^{-64}
	Advanced-III	11,688 (5,884)	404 (40)	11,752	2^{-192}
5	Paranoiac-I	14,754 (7,417)	532 (40)	14,818	2^{-64}
	Paranoiac-IV	17,714 (8,897)	566 (40)	17,778	2^{-256}

Best known classical attack: [CS16] $\rightarrow$ work factor $2^{-2w\log\left(1-\frac{k}{n}\right)(1+o(1))}$ (Prange [Pra62])

Best known quantum attack: ISD with [Gro96] $\rightarrow$ work factor $\sqrt{\binom{n}{2w}/\binom{n-k}{2w}}$

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Pros and cons

Limitations:

- Non-zero decryption failure rate
- Larger ciphertexts than BIKE-1 and BIKE-3 KEMs ($\approx \times 2$)
- Larger public key than BIKE KEM ($\approx \times 2$), but still reasonable

Advantages:

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- **No more hidden trap**
- Makes use of cyclicity for **efficiency**
- Well-understood, theoretically bounded, and fast decreasing DFR
- Attacks on Hamming metric are well understood (50+ years)
- Easy to understand

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